Sustainable Social Systems

or a problem in the **science of combinatorial sociology**

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An **individual** is a person, an animal, a machine, or an element of nature that can be considered distinct from its environment. A **social system**, or simply system, is a set of individuals. Each individual has **needs** which must be satisfied to preserve the individual in an `acceptable` level of existence. Each need is satisfied by **goods**, that are produced by individuals. A system is **self-contained** if the individuals that constitute it use goods produced exclusively by them, while a system is **introvert** if the goods produced by its individuals are consumed only by them. In other words, at a self-contained system there are not imports of goods from outside the system, while at an introvert system there are not exports of goods to outside the system. A system is **closed** if it is *both* self-contained *and* introvert, otherwise it is **open**.

Let $\mathcal{I}$ be the set of individuals that exist in the `real world`, and $\mathcal{S} = 2^{\mathcal{I}}$ the power set of $\mathcal{I}$. The elements of $\mathcal{S}$ are systems, including the empty set $\varnothing$ and the set $\mathcal{I}$. The number of individuals in a system $S \in \mathcal{S}$ is the cardinality of this set and is denoted by $|S|$. Given an individual $I \in \mathcal{I}$, the set of its needs is denoted by $N(I)$, while the set of the goods it produces is denoted by $G(I)$. Correspondingly, for a system $S \in \mathcal{S}$ we have

$$N(S) = \bigcup_{I \in S} N(I)$$

and

$$G(S) = \bigcup_{I \in S} G(I).$$

The fact that a good g satisfies a need n is denoted by $g \succ n$, while if g can't cover n then we write $g \not\succ n$. The notations can be generalized to sets of goods and needs. A system $S \in \mathcal{S}$ is called **sustainable**, or **viable**, if the individuals $I \in S$ satisfy their needs directly by goods produced inside the system. Notice that according to the definition, S is sustainable if *both* $G(S) \succ N(S)$ *and* the needs of individuals $I \in S$ are *actually* satisfied by goods produced by them. The second part of this condition refers to real life and implies a decision, which is taken by the human individuals of a system (we assume, without loss of generality, that machines and nature's elements do not have will), if they want to satisfy their needs with their goods or not, that is, to decide if they want their system to be sustainable or not. Of course, the validity of the first part of the condition gives them the opportunity to choose. A system S where $G(S) \succ N(S)$ is called **potentially sustainable**.

In a sustainable system, the produced quantities of the goods possibly exceed the required quantities to meet the corresponding needs. The surplus may or may not be disposed outside the system, and therefore a sustainable system might be introvert or not. On the other hand, a sustainable system is always self-contained. A self-contained system may or may not be sustainable since all needs of the individuals in a closed system are not necessarily satisfied. In a potentially sustainable system, despite the plenitude of goods, the individuals may decide to import goods from outside the system, and so a potentially sustainable system might be self-contained or not.

A **partially sustainable** system S is a set of individuals that satisfy a *subset* $M \subset N(S)$ of their needs with goods from $G(S)$. In such a system, we know that $G(S) \succ M$, while we don't know the relation between $G(S)$ and $N(S)$ since the strict sustainability does not hold either because it is infeasible, $G(S) \not\succ N(S)$, or by decision.

The next two statements concerning the systems of minimum and maximum cardinality, although they are quite obvious to many scientists, are treated here as assumptions.

Conjecture 1. *The empty system is sustainable, since* $N(\varnothing) = \varnothing$.

Conjecture 2. *The real world, that is the system* $S = \mathcal{I}$*, is not sustainable, since there are individuals having unsatisfied needs.*

It is an open problem whether different assumptions about these two systems, or other well-defined systems, lead to different conclusions. In any case, the basic question is what happens between these two extreme points, and especially what is the situation for the social systems on earth. These questions can be stated by the following problem.

Problem 1. *Given the set* $\mathcal{I}$ *of individuals existing in the real world, find the social systems* $S \in 2^{\mathcal{I}}$ *that are sustainable.*

In Problem 1, the set $\mathcal{I}$ is considered given. In practice, we need to determine, precisely or at least approximately, the characteristics of the individuals that exist in real world, and particularly on earth or in our solar system. Moreover, since the productive capability and the needs −biological/physiological and secondary (subjective, psychological, social, etc.) needs− of each individual change through time, their determined characteristics should be updated too. A partial answer to the question stated in Problem 1 is to find *some* sustainable systems, that is, to find or to *create* sets of individuals that constitute sustainable systems. Probably this will be the case in some future works. Reasonable relaxations to the notions and definitions given above can be done in specific cases, as soon as they don't lead to results practically inapplicable. For example, the sun is a part of many social systems, and it consumes goods, its fuels, that *exist* in the system, but are not *produced* in the system, however we can be quite sure that the conclusions for those systems will not "collapse" for the next few billion years.

Most of the previous clarifications show the dual nature of the current problem, and generally of the projects in **combinatorial sociology**. On one side of this duality, we have the rigorous definitions and procedures of mathematics and computer science, and on the other side we have the practical restrictions of the real world expressed in social and human sciences. The current article proposes to bridge the gap between the two, and suggests a way to implement the powerful and efficient methods from the combinatorial optimization* framework −a well-studied field of applied mathematics and theoretical computer science− to social problems.

A potential design of this bridge can be as follows. On the social part, the actual *data* are collected, recorded, or created. On an intermediate part, the data are modelled in appropriate *data structures*, which are particular ways of representing and organizing the data in a computer so that we can use them efficiently in specific algorithms. On the computer-science part, *mathematical*, *computational*, and *combinatorial analysis* is performed on the data, exactly like any well-defined combinatorial problem (in fully contradiction to the statistical methods), and the output of those algorithms are interpreted into the framework of the real world and the social sciences. A final step after building the bridge is actually to walk it, especially if the output of the algorithmic step is encouraging. In this final step, the *practical implementation* of the corresponding system is applied in real life, and the actual sustainable system exists in reality. In this way, the results are proven *both* from the mathematical and the social perspective, although the proof procedure and the notion of proof itself are completely different in these fields.

*Many complex problems consist in finding optimal solutions (objects) in a large but finite solution space (set of objects) −recall the definitions of social systems, sustainable systems, and Problem 1. Combinatorial optimization is concerned with the study of effective algorithms for solving such problems by cleverly exploring the solution space. It is a subset of mathematical optimization that is related to operations research, algorithm theory, and computational theory.